Lecture - Functional Analysis II
SoSe 2021

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Topics: In this lecture we continue our study of infinite dimensional topological vector spaces and linear operators on them, which we started in Funktionalanalyis I. While the material for the first semester of a functional analysis class is more or less standard, the content of the second semester depends more on the taste of the lecturer. I decided to focus on a variety of fundamental abstract theorems with broad applications in mathematical analysis and beyond (ergodic theory, probability theory, mathematical physics, partial differential equations,...). If the time permits, I plan to cover the following topics:

1. Compact operators on Banach spaces and their basic spectral theory. The spectral theorem for compact normal operators on Hilbert spaces.

2. Fundamental theorems for spaces of continuous functions (Arzela-Ascoli theorem, Stone-Weierstraß theorem, Riesz-Markov-Kakutani representation theorem).

3. Basics on locally convex topological vector spaces and weak topologies.

4. Tychonoff's theorem and compactness theorems in locally convex spaces (Banach-Alaoglu-Bourbaki theorem and Eberlein-Shmulyan theorem).

5. Fundamental theorems in convex analysis (Hahn-Banach separation theorems, bipolar theorem, Krein-Milman theorem, basics on Choquet theory).

Prerequisites: Measure theory and basic knowledge on topics covered in the course Funktionalanalyis I (basics on general topology, Banach spaces and their duals, Hahn-Banach theorem and its consequences for duals of normed spaces, basics on Hilbert spaces, uniform boundedness principle, closed graph and open mapping theorem, basics on spectral theory of closed operators).

Mode: Due to the pandemic the lecture will be held online via BigBlueButton. I prepare detailed notes "the Coronascript", which we read together. Each week there will be one input session (Wednesdays), where I discuss definitions and results without giving detailed proofs. After that all students have until the next session in the following week (Tuesdays) to read

through the material. In that session (Tuesdays) we then discuss proofs and questions from the audience.

Times: Tuesdays 11:15 -12:45 and Wednesdays 15:15 - 16:45 via BigBlueButton. All sessions are recorded and are made available online. Please register for the course in Moodle, where I post the script and the link to BBB.

Exercises: There are no exercise classes for this lecture. However, I included many exercises in the notes and I encourage everyone to try to solve them. We can use part of the Tuesday session to discuss solutions.

Literatur:

1. Manfred Einsiedler and Thomas Ward. *Functional analysis, spectral theory, and applications.* Graduate Texts in Mathematics, 276. Springer, Cham, 2017.

2. Michael Reed and Barry Simon. *Methods of Modern Mathematical Physics I, II and IV.* Academic Press New York and London.

3. Joachim Weidmann. *Lineare Operatoren in Hilberträumen 1: Grundlagen.* Mathematische Leitfäden. B. G. Teubner, Stuttgart, 2000.

4. Dirk Werner. *Funktionalanalysis.* Springer-Verlag, Berlin, 2018.