Workshop
Operator Theoretic Aspects of Ergodic Theory 12
June 7/8th 2024, University of Leipzig

Program

Friday

9.00 - 10.00 Matthias Erbar - *Gradient flows with Dirichlet boundary conditions*

10.00 - 10.30 Florentin Münch - *Markov chain curvature and mixing*

10.30 - 11.00 **Coffee Break**

11.00 - 11.30 Praveen Sharma - *Interpolation Results for Convergence of Implicit Euler Schemes with Accretive Operators*

11.30 - 12.00 Elias Zimmermann - *Exponential mixing and asymptotic equipartition for Gibbs fields on regular trees*

12.00 - 12.30 Noa Bihlmaier - *A categorical approach to representing spaces of measurable maps*

12.30 - 14.00 **Lunch Break**

14.00 - 15.00 Melchior Wirth - *Logarithmic Sobolev Inequalities for Quantum Markov Semigroups*

15.30 - 16.00 Benjamin Hinrichs - *Ergodicity of Feynman-Kac Semigroups on Fock Spaces*

16.00 - 16.30 **Coffee Break**

16.30 - 17.00 Christian Rose - *Sobolev dimensions and heat kernels on graphs*

19.00 - ??.?? **Conference Dinner** - Restaurant Andria

Saturday

9.00 - 10.00 Neil Mañibo - *Ghost measures and the finiteness conjecture for semigroups of matrices*

10.00 - 10.30 Asgar Jamneshan - *The Geometric Representation of Relatively Ergodic Compact Extensions*

10.30 - 11.00 **Coffee Break**

11.00 - 11.30 Micky Bartmann - *Uniform pointwise ergodic theorems for operators*

11.30 - 12.00 Till Hauser - *About mean equicontinuous factor maps*

12.00 - 12.30 Raj Dahya - *Applications of the Bhat–Skeide interpolation to commuting families of operators and $\mathcal{C}_0$-semigroups*

Abstracts

Micky Barthmann
(TU Chemnitz)
Uniform pointwise ergodic theorems for operators

The pointwise theorem of Birkhoff has been generalized in many directions. One direction of generalization has been to consider linear operators that are more general than Koopman operators, such as Dunford-Schwartz operators, i.e., $L^1 - L^\infty$ contractions on a σ-finite measure space. Another direction of generalization is to consider (finite) measure preserving systems that have stronger mixing properties than ergodicity, as was done for example in the Wiener-Wintner Theorem. I will discuss a uniform vector-valued Wiener-Wintner Theorem for a class of operators that includes compositions of ergodic koopman operators and contractive multiplication operators.

This is based on joint work with Sohail Farhangi.

Noa Bihlmaier
(Universität Thübingen)
A categorical approach to representing spaces of measurable maps

We develop a categorical view on classical theorems regarding the representation of spaces of measurable or bounded measurable functions as continuous functions on the canonical model of the measure space. This leads to a general adjunction between a new category of measure spaces and compact Hausdorff spaces, allowing generalizations of classical scalar valued representation results e.g. to the vector valued setting.

This talk is based on a joint project with Asgar Jamneshan.

Raj Dahya
(Universität Leipzig)
Applications of the Bhat–Skeide interpolation to commuting families of operators and $\mathcal{C}_0$-semigroups

In a recent paper [2] we generalised a technique of Bhat and Skeide [1] to be able to interpolate commuting families $\{S_i\}_{i \in \mathcal{I}}$ of contractions on a Hilbert space $\mathcal{H}$, to commuting families $\{T_i\}_{i \in \mathcal{I}}$ of contractive $\mathcal{C}_0$-semigroupson $L^2(\prod_{i \in \mathcal{I}} \mathbb{T}) \otimes \mathcal{H}$ and developed certain 0-1-results.

The Bhat–Skeide interpolation allows us to reduce verification of various properties of continuous processes to those of discrete processes and is thus interesting in and of itself. We consider its application to non-dilatability in continuous-time (*cf.* Parrott's construction [5]), to the generic failure of the von Neumann polynomial inequality (*cf.* [7, 8, 6]), as well as a multi-parameter version of the embedding problem (*cf.* [3, 4]).

References

[1] B. R. Bhat and M. Skeide, *Pure semigroups of isometries on Hilbert C^*-modules*, Journal of Functional Analysis, 269 (2015), pp. 1539–1562.

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[4] ———, *A 'typical' contraction is unitary*, Enseign. Math. (2), 56 (2010), pp. 403–410.

[5] S. Parrott, *Unitary dilations for commuting contractions*, Pacific J. Math., 34 (1970), pp. 481–490.

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[8] ———, *On an inequality of von Neumann and an application of the metric theory of tensor products to operators theory*, J. Functional Analysis, 16 (1974), pp. 83–100.

Matthias Erbar
(Universität Bielefeld)
Gradient flows with Dirichlet boundary conditions

We consider (non-linear) diffusion equations with constant Dirichlet boundary condition in a bounded domain and give a variational characterisation of solutions in terms of gradient flows in the space of measures. While a large body of literature characterising various evlutionary PDEs with Neumann boundary conditions as gradient flow w.r.t. the Wasserstein distance is available, little is known to date concerning other types of boundary conditions. We revisit work by Figalli and Gigli who have introduced a variant of the Wasserstein distance allowing for change of mass by letting the boundary of the domain act as a reservoir. They showed that minimising movements of the entropy wrt this distance converge to solutions of the heat equation with Dirichlet boundary conditions. We give new characterisation of this distance as a dynamic optimal transport problem and rigorously characterise the broader class of non-linear diffusion equations as curves of maximal slope for appropiate entropy functionals.

This is joint work with Giulia Meglioli

Till Hauser
(DLR Köln)
About mean equicontinuous factor maps

In this talk we introduce a notion of mean equicontinuity for factor maps and relate this notion to other possible properties of factor maps, such as distality, proximality, equicontinuity, topo-isomorphy and Banach proximality. We study the setting of actions of discrete amenable groups.

Benjamin Hinrichs
(Universität Paderborn)
Ergodicity of Feynman-Kac Semigroups on Fock Spaces

The ergodicity of contraction semigroups and the spectral theory of their generators are linked by the famous Perron-Frobenius-Faris theorem, which allows to prove non-degeneracy of the lowest eigenvalue in cases where the corresponding eigenspace is far from explicit. We first recall how the ergodicity of Schrödinger semigroups follows easily from the Feynman-Kac formula. We then discuss ergodicity of a class of explicit models from quantum field theory, given by a Schrödinger-type operator with Hilbert space operator-valued potential.

This talk is based on joint work with Fumio Hiroshima and Oliver Matte.

Asgar Jamneshan
(Koç University)
The Geometric Representation of Relatively Ergodic Compact Extensions

We establish that a relatively ergodic extension of arbitrary measure-preserving dynamical systems has a relative discrete spectrum if and only if it is isomorphic to a generalized skew-product by a bundle of compact homogeneous spaces. This result extends previous findings by Mackey, Zimmer, Ellis, Austin, as well as the speaker and Tao. Our innovations include a Peter-Weyl-type representation theory for compact group bundles.

This work is based on a paper with Nikolai Edeko and Henrik Kreidler.

Neil Mañibo
(Universität Bielefeld)
Ghost measures and the finiteness conjecture for semigroups of matrices

In 1995, Lagarias and Wang asked whether, given a finite set S of real matrices, there always exists a finite product of matrices from S that realises the joint spectral radius of S. The general conjecture has been shown to be false for real matrices, but it remains open for matrices with rational/integer entries. Regular sequences, which are a generalisation of automatic sequences, are intimately related to this question. In this talk, we will discuss how to build probability measures on [0,1) from regular sequences, and show that for a specific class of matrices, the finiteness property holds by confirming an equivalent spectral property for the derived measures.

This is based on joint work with Michael Coons, James Evans, and Philipp Gohlke.

Florentin Münch
(MPI MiS Leipzig)
Markov chain curvature and mixing

We show that the Ollivier Ricci curvature of a Markov chain controls the log-Sobolev constant. In case of non-negative Ollivier sectional curvature, the log-Sobolev constant can be lower bounded by the minimum Ollivier Ricci curvature. By this, we answer an open question by Peres and Tetali. In case of non-negative Ricci curvature, the log-Sobolev constant can be lower bounded in terms of the diameter. Moreover in case of non-negative Ollivier Ricci curvature, we give an upper bound for the spectral gap in terms of the mixing time. This gives a quantitative negative answer to the question by Naor and Milman, whether there can be expander graphs with non-negative curvature.

Christian Rose
(Universität Potsdam)
Sobolev dimensions and heat kernels on graphs

On Riemannian manifolds it is known that Sobolev inequalities in balls are equivalent to the conjunction of Gaussian upper bounds and volume doubling. On graphs, this equivalence is expected to hold on large balls. In the case of the normalizing measure, i.e., bounded Laplacian, it turns out that an additional regularity condition on the measure is necessary for the equivalence to hold. If a generalization and unification to arbitrary measure, i.e., possibly unbounded Laplacians, is desired, a new local regularity condition enters the equivalence naturally. Moreover, the dimension of Sobolev inequalities in balls has to vary and depends on the maximal vertex degree inside the ball. It converges if the vertex degree does not grow too fast.

This is joint work with Matthias Keller.

Praveen Sharma
(University of Delhi)
Interpolation results for convergence of implicit Euler schemes with accretive operators

Consider the nonlinear abstract Cauchy problem

$$(\text{CP}\colon f, u^0) \qquad \begin{cases} \dot{u}(t) + Au(t) \ni f(t) & (t \in [0, T]), \\ u(0) = u^0, \end{cases}$$

with an m-accretive operator $A \subseteq X \times X$ of some type $\omega \in \mathbb{R}$ on a Banach space X, $T > 0$, $f \in L^1(0, T; X)$ and $u^0 \in \overline{\operatorname{dom} A}$ It is also well known that if $u^0 \in \operatorname{dom} A$ and $f \in BV(0, T; X)$, then Euler solutions are not just continuous but Lipschitz continuous (see Benilan, Crandall and Pazy [1, Lemma 7.8]). Moreover, in this case, solutions of appropriate Euler schemes converge with the rate $O(|\pi|^{1/2})$ to the actual solution, where $|\pi|$ is the

mesh size of the time partition π (see Nochetto and Savaré [5], recently also Beurich [2], who does not assume a solution to exist but rather estimates the difference of two approximate solutions). The corresponding estimate generalizes an explicit estimate by Kobayashi [4] in the case when $f = 0$, and the speed of convergence seems to be optimal even for linear Cauchy problems.

In this article, we extend both results (regularity of the solution, speed of convergence of the implicit Euler scheme) to the case when u^0 and f belong to certain interpolation spaces or interpolation sets between $\mathrm{dom}\,A$ and $\overline{\mathrm{dom}\,A}$ and $BV(0,T;X)$ and $L^1(0,T;X)$ respectively.

This contribution is co-authored by Johann Beurich.

References

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Melchior Wirth
(ISTA Klosterneuburg)
Logarithmic Sobolev Inequalities for Quantum Markov Semigroups

Quantum Markov semigroups model the time evolution of certain open quantum systems. A natural question in this context concerns the long-time behavior of these systems, which often exhibit a return to equilibrium. If the distance from equilibrium is measured in relative entropy, then exponential decay estimates are related to modified logarithmic Sobolev inequalities. This relation is well-studied for finite-dimensional quantum systems, where modified logarithmic Sobolev inequalities have received a lot of attention recently in the context of quantum many-body systems and quantum information theory. In this talk I will present an equivalence between exponential decay of relative entropy and the modified logarithmic Sobolev inequality in a general framework that includes infinite-dimensional quantum systems with equilibrium states that are not Gibbs states. In the final part, I will discuss sufficient conditions for exponential decay of relative entropy inspired by optimal transport and curvature bounds.

This talk is based on joint work with Martijn Caspers, Florentin Münch, Matthijs Vernooij and Haonan Zhang.

Elias Zimmermann
(Universität Leipzig)

Exponential mixing and asymptotic equipartition for Gibbs fields on regular trees

Consider a stationary ergodic process with discrete state space. The asymptotic equipartition property (AEP) states that most of the probability mass is distributed more or less evenly among all cylinder sets specified on a sufficiently large interval. The AEP is a consequence of the Shannon-McMillan-Breiman (SMB) theorem, which plays a key role in information and entropy theory. Due to work of Ornstein, Weiss, Lindenstrauss and many others the latter has been extended to a broad class of random fields over graphs with amenable geometry, such as $\mathbb{Z}^d$ or more generally Cayley graphs of polynomial volume growth.

However, for random fields on non-amenable graphs, asymptotic equipartition is much less understood. In this talk I will focus on regular trees T and explain how one can use an exponential mixing condition, which is instantiated by a large class of Gibbs measures, to obtain an SMB type theorem along spheres in this case. A central building block of the proof is the fact that the boundary ∂T of T carries an amenable structure, which allows to utilizie arguments from the amenable world in this setting.

The talk is based in joint work in progress with Felix Pogorzelski.